

ON THE ANOMALOUS INCREASE OF THE LUNAR ECCENTRICITY

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Possible explanations of the recently reported anomalous increase of the eccentricity of the lunar orbit are sought in terms of classical Newtonian mechanics, general relativity, and long-range modifications of gravity.

Anderson and Nieto, in a recent review¹ of some astrometric anomalies detected in the solar system by some independent groups, mentioned also an anomalous secular increase of the eccentricity e of the orbit of the Moon

$$\dot{e}_{\text{meas}} = (9 \pm 3) \times 10^{-12} \text{ yr}^{-1} \quad (1)$$

based on an analysis of a long LLR data record spanning 38.7 yr performed by Williams and Boggs² with the dynamical force models of the DE421 ephemerides^{3,4} including all the known relevant Newtonian and Einsteinian effects. Notice that Eq. 1 is statistically significant at a 3σ -level. The first account⁵ of this effect appeared in 2001 by Williams *et al.*, who gave an extensive discussion of the state-of-the-art in modeling the tidal dissipation in both the Earth and the Moon. Later, Williams and Dickey⁶, relying upon the 2001 study⁵, released an anomalous eccentricity rate as large as $\dot{e}_{\text{meas}} = (1.6 \pm 0.5) \times 10^{-11} \text{ yr}^{-1}$. Anderson and Nieto¹ commented that Eq. 1 is not compatible with present, standard knowledge of the dissipative processes in the interiors of both the Earth and Moon, which were, actually, modeled by Williams and Boggs².

Naive, dimensional evaluations of the effect caused on e by an additional anomalous acceleration A can be made by noticing that

$$\dot{e} \simeq \frac{A}{na}, \quad (2)$$

with

$$na = 1.0 \times 10^3 \text{ m s}^{-1} = 3.2 \times 10^{10} \text{ m yr}^{-1} \quad (3)$$

for the geocentric orbit of the Moon, whose mass is denoted as m . In it, a is the orbital semimajor axis, while $n \doteq \sqrt{\mu/a^3}$ is the Keplerian mean motion in which $\mu \doteq GM(1 + m/M)$ is the gravitational parameter of the Earth-Moon system: G is the Newtonian constant of gravitation and M is the mass of the Earth. It turns out that an extra-acceleration as large as

$$A \simeq 3 \times 10^{-16} \text{ m s}^{-2} = 0.3 \text{ m yr}^{-2} \quad (4)$$

would satisfy Eq. 1. In fact, a mere order-of-magnitude analysis based on Eq. 2 would be inadequate to infer meaningful conclusions: finding simply that this or that dynamical effect

induces an extra-acceleration of the right order of magnitude may be highly misleading. Indeed, exact calculations of the secular variation of e caused by such putative promising candidate extra-accelerations A must be performed with standard perturbative techniques in order to check if they, actually, cause an averaged non-zero change of the eccentricity. Moreover, it may well happen, in principle, that the resulting analytical expression for $\langle \dot{e} \rangle$ retains multiplicative factors $1/e^j, j = 1, 2, 3, \dots$ or $e^j, j = 1, 2, 3, \dots$ which would notably alter the size of the found non-zero secular change of the eccentricity with respect to the expected values according to Eq. 2.

It is well known that a variety of theoretical paradigms^{7,8} allow for Yukawa-like deviations⁹ from the usual Newtonian inverse-square law of gravitation. The Yukawa-type correction to the Newtonian gravitational potential $U_N = -\mu/r$, where $\mu \doteq GM$ is the gravitational parameter of the central body which acts as source of the supposedly modified gravitational field, is

$$U_Y = -\frac{\alpha\mu_\infty}{r} \exp\left(-\frac{r}{\lambda}\right), \quad (5)$$

in which μ_∞ is the gravitational parameter evaluated at distances r much larger than the scale length λ . In order to compute the long-term effects of Eq. 5 on the eccentricity of a test particle it is convenient to adopt the Lagrange perturbative scheme¹⁰. In such a framework, the equation for the long-term variation of e is¹⁰

$$\left\langle \frac{de}{dt} \right\rangle = \frac{1}{na^2} \left(\frac{1-e^2}{e} \right) \left(\frac{1}{\sqrt{1-e^2}} \frac{\partial \mathcal{R}}{\partial \omega} - \frac{\partial \mathcal{R}}{\partial \mathcal{M}} \right), \quad (6)$$

where ω is the argument of pericenter, \mathcal{M} is the mean anomaly of the test particle, and \mathcal{R} denotes the average of the perturbing potential over one orbital revolution. In the case of a Yukawa-type perturbation, Eq. 5 yields

$$\langle U_Y \rangle = -\frac{\alpha\mu_\infty \exp\left(-\frac{a}{\lambda}\right)}{a} I_0\left(\frac{ae}{\lambda}\right), \quad (7)$$

where $I_0(x)$ is the modified Bessel function of the first kind $I_q(x)$ for $q = 0$. An inspection of Eq. 6 and Eq. 7 immediately tells us that there is no secular variation of e caused by an anomalous Yukawa-type perturbation.

The size of the general relativistic Lense-Thirring¹¹ acceleration experienced by the Moon because of the Earth's angular momentum¹² $S = 5.86 \times 10^{33} \text{ kg m}^2 \text{ s}^{-1}$ is just

$$A_{LT} \simeq \frac{2vGS}{c^2 a^3} = 1.6 \times 10^{-16} \text{ m s}^{-2} = 0.16 \text{ m yr}^{-2}, \quad (8)$$

i.e. close to Eq. 4. On the other hand, it is well known that the Lense-Thirring effect does not cause long-term variations of the eccentricity. Indeed, the integrated shift of e from an initial epoch corresponding to f_0 to a generic time corresponding to f is¹³

$$\Delta e = -\frac{2GS \cos I' (\cos f - \cos f_0)}{c^2 na^3 \sqrt{1-e^2}}, \quad (9)$$

in which I' is the inclination of the Moon's orbit with respect to the Earth's equator and f is the true anomaly. From Eq. 9 it straightforwardly follows that after one orbital revolution, i.e. for $f \rightarrow f_0 + 2\pi$, the long-term gravitomagnetic shift of e vanishes.

A promising candidate for explaining the anomalous increase of the lunar eccentricity is, at least in principle, a trans-Plutonian massive body X of planetary size located in the remote peripheries of the solar system. Indeed, the perturbation induced by it would, actually, cause a

non-vanishing long-term variation of e . Moreover, since it depends on the spatial position of X in the sky and on its tidal parameter

$$\mathcal{K}_X \doteq \frac{Gm_X}{d_X^3}, \quad (10)$$

where m_X and d_X are the mass and the distance of X, respectively, it may happen that a suitable combination of them is able to reproduce Eq. 1. Let us recall that, in general, the perturbing potential felt by a test particle orbiting a central body due to a very distant, pointlike mass can be cast into the following quadrupolar form

$$U_X = \frac{\mathcal{K}_X}{2} \left[r^2 - 3 \left(\vec{r} \cdot \hat{l} \right)^2 \right], \quad (11)$$

where $\hat{l} = \{l_x, l_y, l_z\}$ is a unit vector directed towards X determining its position in the sky. In Eq. 11 $\vec{r} = \{x, y, z\}$ is the geocentric position vector of the perturbed particle, which, in the present case, is the Moon. Iorio¹⁴ has recently shown that the average of Eq. 11 over one orbital revolution of the particle is

$$\langle U_X \rangle = \frac{\mathcal{K}_X a^2}{32} \mathcal{U} \left(e, I, \Omega, \omega; \hat{l} \right), \quad (12)$$

where $\mathcal{U} \left(e, I, \Omega, \omega; \hat{l} \right)$ is a complicated function of its arguments¹⁴: Ω is the longitude of the ascending node and I is the inclination of the lunar orbit to the ecliptic. In the integration yielding Eq. 12 \hat{l} was kept fixed over one orbital revolution of the Moon, as it is reasonable given the assumed large distance of X with respect to it. Eq. 6, applied to Eq. 12, straightforwardly yields

$$\langle \dot{e} \rangle = \frac{15\mathcal{K}_X e \sqrt{1-e^2}}{16n} \mathcal{E} \left(I, \Omega, \omega; \hat{l} \right). \quad (13)$$

Also $\mathcal{E} \left(I, \Omega, \omega; \hat{l} \right)$ is an involved function of the orientation of the lunar orbit in space and of the position of X in the sky¹⁴. Actually, the expectations concerning X are doomed to fade away. Indeed, apart from the modulation introduced by the presence of the time-varying I, ω and Ω in Eq. 13, the values for the tidal parameter which would allow to obtain Eq. 1 are too large for all the conceivable positions $\{\beta_X, \lambda_X\}$ of X in the sky. This can easily be checked by keeping ω and Ω fixed at their J2000.0 values as a first approximation. Indeed, Iorio¹⁴ showed that the physical and orbital features of X postulated by two recent plausible theoretical scenarios^{15,16} for X would induce long-term variations of the lunar eccentricity much smaller than Eq. 1. Conversely, it turns out that a tidal parameter as large as

$$\mathcal{K}_X = 4.46 \times 10^{-24} \text{ s}^{-2} \quad (14)$$

would yield the result of Eq. 1. Actually, Eq. 14 is totally unacceptable since it corresponds to distances of X as absurdly small as $d_X = 30$ au for a terrestrial body, and $d_X = 200$ au for a Jovian mass.

An empirical explanation of Eq. 1 can be found by assuming that, in addition to the usual Newtonian inverse-square law for the gravitational acceleration imparted to a test particle by a central body orbited by it, there is also a small radial extra-acceleration of the form

$$A = k H_0 v_r. \quad (15)$$

In it k is a positive numerical parameter of the order of unity to be determined from the observations, $H_0 = (73.8 \pm 2.4) \text{ km s}^{-1} \text{ Mpc}^{-1} = (7.47 \pm 0.24) \times 10^{-11} \text{ yr}^{-1}$ is the Hubble

parameter at the present epoch¹⁷, defined in terms of the time-varying cosmological scaling factor $S(t)$ as $H_0 \doteq \dot{S}/S|_0$, and v_r is the component of the velocity vector \vec{v} of the test particle's proper motion about the central body along the common radial direction. Indeed, a straightforward application of the Gauss perturbative equation for e to Eq. 15 yields

$$\langle \dot{e} \rangle = kH_0 \frac{(1 - e^2) \left(1 - \sqrt{1 - e^2}\right)}{e}. \quad (16)$$

Since $e_{\text{Moon}} = 0.0647$, Eq. 16 can reproduce Eq. 1 for $2.5 \lesssim k \lesssim 5$. Here we do not intend to speculate too much about possible viable physical mechanisms yielding the extra-acceleration of Eq. 15. It might be argued that, reasoning within a cosmological framework, the Hubble law may give Eq. 15 for $k = 1$ if the proper motion of the particle about the central mass is taken into account in addition to its purely cosmological recession which, instead, yields the well-known local extra-acceleration of tidal type $A_{\text{cosmol}} = -q_0 H_0^2 r$, where q_0 is the deceleration parameter at the present epoch.

Acknowledgments

I gratefully acknowledge the financial support by the MORIOND scientific committee

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